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Engineering Notes

Range and Endurance Estimates for Battery-Powered Aircraft

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Introduction

THE utility and cost effectiveness of small unmanned aerial vehicles (UAVs) has seen a large increase in their use, in both civilian and military applications. Depending on the particular requirements, the aircraft may be powered using a piston-gasoline engine or an electric motor. Electric propulsion appears to be favored as the UAV size diminishes or if stealth, in terms of acoustic signature, is a design requirement.

Expressions to estimate the range and endurance of piston propeller and jet aircraft are well established [1,2] (the Breguet equations). Estimates for the range and endurance of electric aircraft are less well established [3,4] and may not be presented in a fashion consistent with that typically employed in the aeronautics community. Consideration of the propulsion system would suggest that equating the power delivered by the battery, accounting for losses due to the propeller, motor, and motor controller, to the power required to overcome drag would yield performance estimates. While this approach is sound, the behavior of a battery and its effective capacity, depending on the current draw (the so-called Peukert effect [5]), should be accounted for; otherwise, significant estimation errors may occur.

Consequently, in this Note, expressions are derived to estimate the range and endurance of a battery-powered electric aircraft, accounting for battery discharge behavior. The impact of ignoring the Peukert effect is investigated. Parameters affecting performance are examined.

Theoretical Development

For an aircraft in steady level flight, the power required $P\text{req}$ to overcome the drag D of the plane at a given flight velocity U is given by

$$P\text{req} = D \times U \tag{1}$$

The drag polar for an aircraft with a moderately cambered wing is commonly approximated by (assuming incompressible flow)

$$C_D = C_{Do} + kC_L^2 \tag{2}$$

where C_{Do} is the zero lift drag and is dominated by viscous effects, essentially shear, and thus skin friction drag. The kC_L^2 term relates to the lift-dependent drag comprising vortex drag and viscous pressure drag. Equation (2) may be redimensionalized by multiplication by the dynamic pressure q and reference area S, yielding

$$D = qS(C_{Do} + kC_L^2) \tag{3}$$

Assuming steady level flight requires the equality of the aircraft's lift L and weight W. Substituting $L = W = 0.5\rho U^2 SC_L$ into Eq. (3) (with $\rho =$ density and $C_L =$ lift coefficient) and then Eq. (1) results in

$$P\text{req} = \frac{1}{2}\rho U^3 SC_{Do} + \frac{2W^2 k}{\rho US} \tag{4}$$

For an electric aircraft considered in this Note, the power required to overcome the drag is provided by the battery pack. The capacity of a battery is typically quoted in ampere hours or milliampere hours. Thus, a battery with a capacity of 2 Ah would appear to be capable of supplying a current of 2 A for 1 h (battery capacity is typically based upon a 1 h discharge for portable batteries or 20 h for larger installations). If the required current where 4 A, however, the battery would not be capable of supplying this output for a half hour, a consequence of the so-called Peukert effect [5]. In essence, the higher the current draw, the less the effective battery capacity. Similarly, if the current draw was 1 A, the battery might show an increase in effective capacity over 2 Ah. Peukert's equation may be written as

$$t = \frac{C}{i^n} \tag{5a}$$

where t is the time in hours, i is the discharge current (amperes), and C is the battery capacity in ampere hours. A discharge parameter dependent on the battery type and temperature is n. Unfortunately, this parameter typically changes for a given battery as it ages and cycles such that capacity usually diminishes. Equation (5a) however, is only valid if the battery is discharged at 1 A, which is seldom the case. A modification to this equation†, accounting for the effect of the discharge rate, is given by

$$t = \frac{Rt}{i^n}\left(\frac{C}{Rt}\right)^n \tag{5b}$$

Rt is the battery hour rating (in hours): i.e., the discharge time over which the capacity was determined (typically 1 h for small rechargeable battery packs). For a battery, the output power may be estimated as (where V is volts)

$$P_B = Vi \tag{6}$$

Substitution of Eq. (5b) into Eq. (6) yields

$$P_B = V\frac{C}{Rt}\left(\frac{Rt}{t}\right)^{1/n} \tag{7}$$

The power output of the battery will be reduced by losses in the propulsion system consisting of the motor driver, motor, and propeller. While each individual element has its own efficiency, for the purposes of this analysis, they will be combined into a total efficiency η_{tot}. Incorporation of η_{tot} and equating Eqs. (4) and (7) gives

$$\left(\frac{Rt}{t}\right)^{1/n}\left(\frac{C}{Rt}\right) = \frac{1}{\eta_{\text{tot}} V}\left[\frac{1}{2}\rho U^3 SC_{Do} + \frac{2W^2 k}{\rho US}\right] \tag{8}$$

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†Data available at http://www.smartgauge.co.uk/peukert2.html [retrieved 20 February 2010].

Solving for the time t results in

$$E = t = Rt^{1-n}\left[\frac{\eta_{\text{tot}} V \times C}{\frac{1}{2}\rho U^3 S C_{Do} + (2W^2 k/\rho U S)}\right]^n \tag{9}$$

where E is the endurance in hours. Equation (9) may be used to estimate the endurance of a battery-powered electric aircraft accounting for the discharge rate of the battery for any achievable flight velocity. As the propulsion system's output is power, classical relations for minimum power and thrust [1,2] may be used to estimate the maximum range and endurance. The condition for maximum endurance is given by

$$C_{Do} = \tfrac{1}{3} k C_L^2 \tag{10a}$$

and the range is given by

$$C_{Do} = k C_L^2 \tag{10b}$$

Equation (10), in conjunction with $L = W = 0.5\rho U^2 S C_L$, gives the required flight velocity for maximum range U_R and endurance U_E as

$$U_E = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{k}{3C_{Do}}}} \tag{11}$$

$$U_R = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{k}{C_{Do}}}} \tag{12}$$

For a given aircraft, the parameters in Eqs. (11) and (12) may be determined from the vehicle's configuration as well as the linearized drag polar, where k is the slope of the linear extent of the plot and C_{Do} is the zero lift axis intercept. Variables in Eq. (9) would be known from the battery and motor specifications. Batteries such lithium–ion, nickel–cadmium, and nickel–metal hydride show a small variation of voltage with discharge [6] until virtually fully discharged.

To calculate the maximum endurance, the required flight velocity is calculated using Eq. (11). This value is then used in Eq. (9). To estimate the maximum range, the required velocity is determined using Eq. (12). Substitution of this value into Eq. (9) gives the time aloft that would yield maximum range. The range is then calculated using

$$R = E \times U_R \tag{13}$$

The efficiency η_{tot} of the propulsion system (propeller and electrical components) may be estimated experimentally through wind-tunnel testing. Power available (thrust multiplied by velocity) may be mapped for different throttle and wind-tunnel velocity settings. Simultaneous measurement of the current and voltage provided by the battery yields the electrical power input. The efficiency η_{tot} can then be determined as the ratio of the available power divided by the electrical power. Alternatively, the efficiency of the individual elements may be estimated using the approaches in [4,7].

Concise expressions specifically for maximum range and endurance may be established using classical results for flight at minimum power and thrust [Eq. (10)]. For maximum endurance, flight at minimum power is desired such that $C_{Do} = 1/3kC_L^2$. Substitution of this relation into

$$P\text{req} = \tfrac{1}{2}\rho U^3 S (C_{Do} + k C_L^2) \tag{14}$$

with $C_L = \sqrt{(3C_{Do}/k)}$ and $L = W = 0.5\rho U^2 S C_L$ for steady level flight yields

$$P\text{req} = \frac{2}{\sqrt{\rho S}} C_{Do}^{1/4}\left(2W\sqrt{\frac{k}{3}}\right)^{3/2} \tag{15}$$

Substitution of Eq. (15) into Eq. (9), noting that the denominator in the bracketed term in Eq. (9) is the required power, gives the maximum endurance $E_{\max}$ as

$$E_{\max} = Rt^{1-n}\left(\frac{\eta_{\text{tot}} V \times C}{(2/\sqrt{\rho S}) C_{Do}^{1/4} (2W\sqrt{k/3})^{3/2}}\right)^n \text{ h} \tag{16}$$

A similar approach for maximum range $R_{\max}$, noting that for the required minimum thrust $C_{Do} = kC_L^2$ and $C_L = \sqrt{(C_{Do}/k)}$, yields

$$P\text{req} = \frac{1}{\sqrt{\rho S}} C_{Do}^{1/4} (2W\sqrt{k})^{3/2} \tag{17}$$

Substitution of Eq. (17) with Eq. (12) into Eq. (9) results in

$$R_{\max} = Rt^{1-n}\left(\frac{\eta_{\text{tot}} V \times C}{(1/\sqrt{\rho S}) C_{Do}^{1/4} (2W\sqrt{k})^{3/2}}\right)^n \sqrt{\frac{2W}{\rho S}\sqrt{\frac{k}{C_{Do}}}} \cdot 3.6 \text{ km} \tag{18}$$

Equations (16) and (18) may be used to estimate the maximum range and endurance of a battery-powered aircraft. Examination of Eqs. (16) and (18) shows that maximization of E and R is promoted by higher battery/motor voltage (reduced current draw) and battery capacity. Altitude reduces endurance, as the dependence on density is given by $\rho^{n/2}$. Range is weakly affected by altitude with a $\rho^{(n-1)/2}$ dependency.

To explicitly investigate the impact of the discharge rate on effective battery capacity, simulations were performed using $n = 1$ (an ideal battery: no Peukert effect) and $n = 1.3$ (typical for lithium–polymer batteries). Additionally, set or rated battery capacities of 1, 2, 3, and 4 Ah were examined. Required input data were taken from [8], as it contains detailed measurements for an electric battery-powered UAV. Consequently, the following input values were used for all presented plots, unless otherwise mentioned: $\eta_{\text{tot}} = 0.5$, $\rho = 1.2\ \text{kg/m}^3$, $Rt = 1$ h, $W = 9.34$ N, $S = 0.32\ \text{m}^2$, $V = 11.1$ V, $C_{Do} = 0.015$, and $k = 0.13$.

Figure 1 presents the effect of battery capacity and n on the calculated range and endurance for a fixed total weight (as may be effected through improvements in future battery technology). As seen, the Peukert effect is significant and can be favorable if the battery capacity is large with respect to the current draw: i.e., a 33% increase in endurance and 29% increase in range for a 4 Ah capacity battery pack, compared with an ideal battery. As the rated battery capacity reduces, the Peukert effect attenuates performance, as the battery cannot recover from the current drain. For a capacity of 1 Ah, endurance and range is reduced by 12.5 and 15.1%, respectively, when battery effects are included. Even for high battery capacity, the effect can reverse (i.e., from increasing performance) if the current draw becomes significant (i.e., high flight velocity), as indicated by the crossover points in Fig. 1. The associated current draw is included in the top plot of Fig. 1 for reference. As seen, the crossover point corresponds to required current, equaling set battery capacity.

Figure 2 summarizes the effect of n on maximum range and endurance as a function of rated battery capacity for a fixed total weight. The presented curves are the ratio of $n = 1.3$ over $n = 1$ for range and endurance. At these conditions, current draw was 1.56 A for endurance and 1.78 A for range. Consequently, the effective battery capacity is enhanced for presented capacities of two and higher. Specifically, a capacity of 1.56 Ah would yield an ideal battery (n is effectively one) at maximum endurance conditions; similarly, $C = 1.78$ Ah at maximum range conditions, as is also implied by Eq. (5b). The form of the curves suggests that increasing effective capacity benefits both range and endurance stemming from the Peukert effect but to a lesser relative extent as C increases (observe the decreasing slope of the curves). Note that effective battery capacity is also dependent on temperature, such that increasing temperature (within bounds) generally improves capacity [9].

Figure 1 is indicative of the impact of improved battery technology, allowing greater capacity for a fixed battery weight. However, an increase in battery capacity usually requires the addition

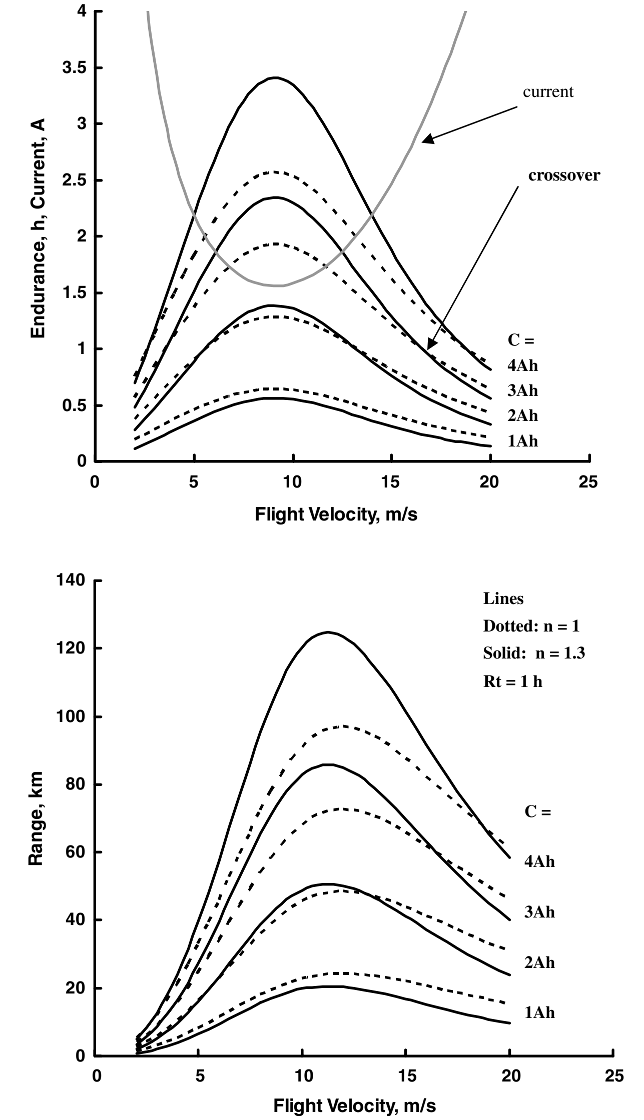

Fig. 1 Effect of battery capacity on estimated endurance [Eq. (9)] and range [Eq. (13)], assuming constant gross weight.

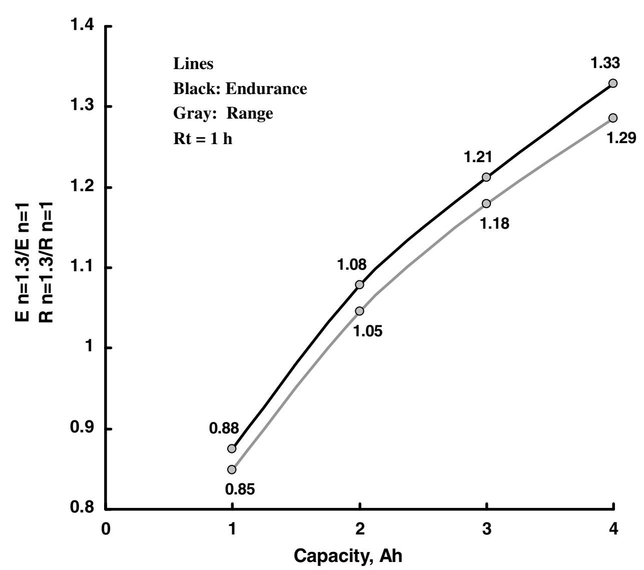

Fig. 2 Effect of *n* parameter on maximum range and endurance, assuming constant gross weight.

of cells; thus, the weight of the plane increases. Increasing the number of cells generally increases the weight of supporting systems (e.g., structure). Consequently, to assess the potential impact of capacity accounting for this cyclical effect, the weight of the battery pack is set as an assumed fraction of the total aircraft weight. Thus, a capacity increase will also increase the weight of the rest of the aircraft by a proportional amount. While this explicit scaling may not be entirely representative, it is suitable for demonstrative purposes.

To account for additional cells, the total capacity of the battery pack C_{tot} may be expressed as

$$C_{\text{tot}} = jC_{\text{batt}} \tag{19}$$

C_{batt} is the capacity of each battery, and the UAVs total weight W_{tot} may be expressed as

$$W_{\text{tot}} = \frac{j \times W_{\text{batt}}}{BR} \tag{20}$$

where BR is the battery weight as a fraction of the total weight (e.g., typically 0.3 to 0.4), W_{batt} is the weight of each individual battery, and j is a counter expressing the number of batteries. Equations (19) and (20) may be substituted into Eqs. (16) and (18) for C and W, respectively. For the aircraft data in [8], the battery weight was estimated using the average weight for similarly rated lithium–polymer batteries from numerous Internet vendors, as the specific battery was not cited in the reference. Consequently, a battery weight W_{batt} of 0.8 N was used with $C_{\text{batt}} = 1.5$ Ah at 11.1 V, corresponding to a Mystery battery pack. This gives a specific energy of $208\ \text{W} \cdot \text{h/kg}$.

Figure 3 shows the effect of battery capacity and corresponding weight increase for BR ranging from 0.3 to 0.4. In these estimates, it is assumed that the aircraft's weight increases; however, the geometric dimensions (e.g., wing geometry) remain fixed. As may be seen, although counterintuitive, adding capacity reduces range and endurance for a fixed BR. The increase in total weight that accompanies an increase in capacity results in a higher required flight velocity, required power, and as shown, current draw. A linear increase in capacity results in a nonlinear increase in required current. For a given installed capacity, increasing the battery weight fraction BR is beneficial, as the total aircraft weight reduces. The array of data in Fig. 3 is included to be illustrative; a flight vehicle of a 2 N weight and the imposed fixed geometry would not be practical and would leave little, if any, available capacity for payload. Using the assumed variation of total weight with capacity, Eq. (20) indicates that the endurance is proportional to $j^{-n/2}$ and the range is dependent on $j^{0.5(1-n)}$. Consequently, the endurance shows a much greater sensitivity to capacity. An ideal battery, $n = 1$, indicates that range becomes independent of capacity [at least for the assumed weight dependency; see Eq. (20)].

It was stated earlier that, for most batteries used in UAV applications, voltage drop during discharge is moderate. However, if potentially greater accuracy is required, voltage drop effects may be incorporated. An approach for implementing variable voltage is enumerated below:

1) Calculate the Preq for endurance and/or range [Eq. (15) and/or Eq. (17), respectively].

2) For this required power, calculate the required battery supply current i_o at the rated full-capacity battery supply voltage V_o:

$$i_o = \frac{P\text{req}}{V_o \eta_{\text{tot}}} \tag{21}$$

3) For this i_o, calculate the effective initial battery capacity C_o [given by Eq. (5b) multiplied by i_o]:

$$C_o = i_o^{1-n} Rt^{1-n} C^n \tag{22}$$

where C is the rated total battery capacity for the specified discharge time Rt.

4) The following equation set is then marched in time, using specified time increments of Δt and incrementing time using $j = 0, 1, 2, 3, \ldots, N$, with time $= t = j\Delta t$.

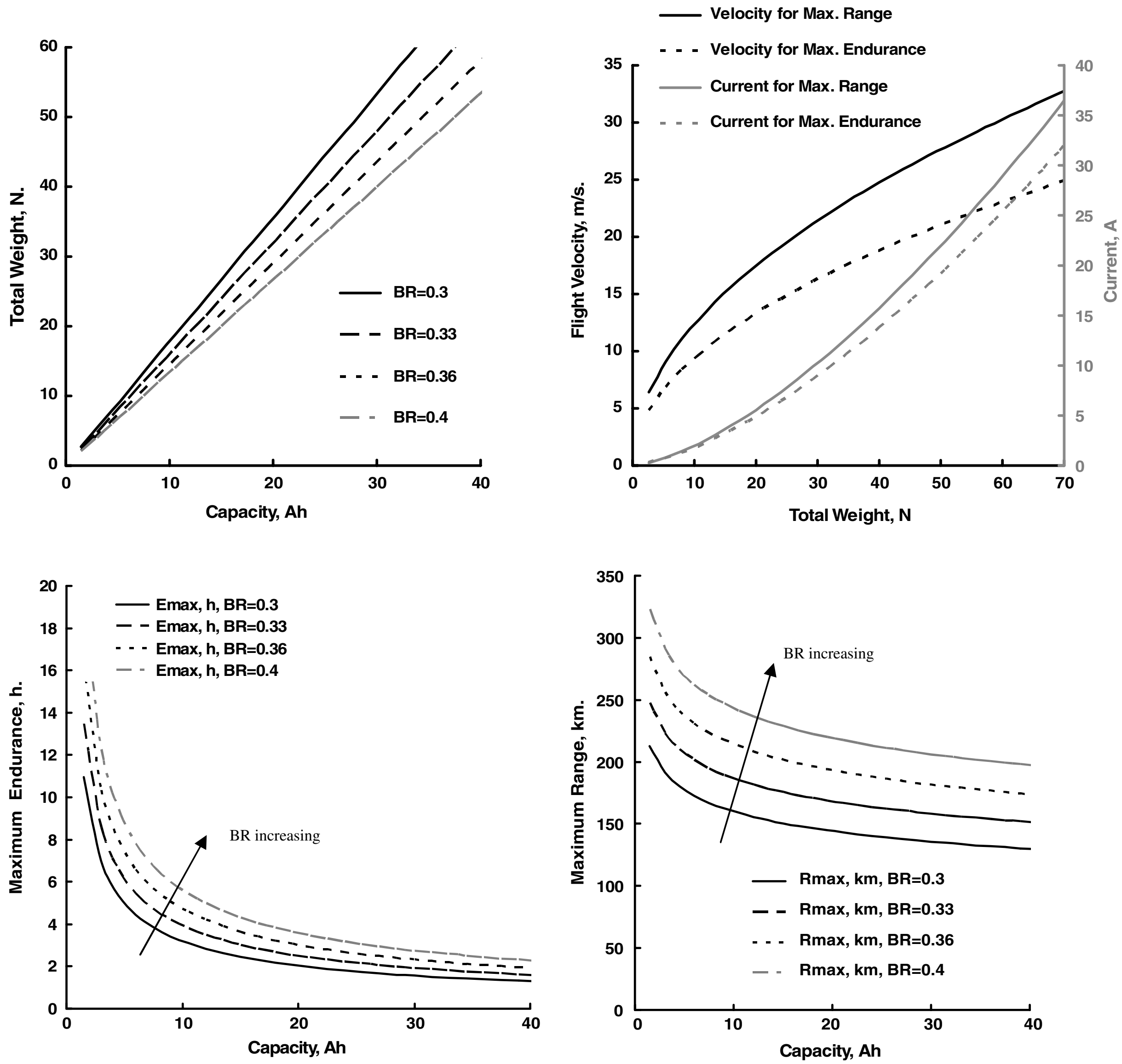

Fig. 3 Effect of increasing battery capacity on weight, maximum range, endurance, flight velocity, and current draw $n = 1.3$.

5) The estimated battery voltage drop during discharge may be estimated from the manufacturer's data, or it may be determined experimentally. Most constant load discharges show a small linear reduction in voltage as the battery discharges. This may be simulated using

$$V_{j+1}(t) = V_o - k1[C_o - C_j(t)] \tag{23}$$

where $C_j(t)$ is the battery's effective capacity at each time step and is equal to C_o for $j = 0$. The parameter $k1$ is, effectively, the slope of the linear curve fit represented by Eq. (23) and can be simply adjusted to give a desired voltage drop.

6) As the voltage drops with time, this requires the battery supply current to increase to maintain the same power. The required current may be found using

$$i_{j+1}(t) = \frac{P\text{req}}{V_{j+1}(t)\eta_{\text{tot}}} \tag{24}$$

7) The remaining battery capacity C_{j+1} may then be estimated using

$$C_{j+1}(t) = i_{j+1}(t)^{1-n} R t^{1-n} C^n - \sum_{m=1}^{j+1} i_m(t)\Delta t \tag{25}$$

This relation gives the effective battery capacity based on the current draw at this time step less the total capacity that has been discharged. During each iteration, the value for $C_{j+1}(t)$ is then substituted for $C_j(t)$ in Eq. (23).

8) Steps 4 through 7 are repeated until $C_{j+1}(t) \approx 0$, indicating that the battery is discharged, or until a user-specified remaining capacity is reached. At this iteration, the maximum endurance $E_{\max}$ is given by $E_{\max} = j\Delta t$ if Eq. (15) was used for Preq. To calculate the maximum range, Eq. (17) is used for Preq. The endurance, as given by $j\Delta t$ when $C_{j+1}(t) \approx 0$, is then multiplied by $3.6U_R$ to yield the maximum range in kilometers. U_R is given by Eq. (12).

Note that the procedure outlined previously is equivalent to using Eqs. (16) and (18) with $k1 = 0$ (i.e., the voltage does not change during discharge). Figure 4 examines the impact of voltage variation on maximum range and endurance. As an aid to interpretation, the range and endurance at 10 V would indicate that the battery supply voltage was adjusted (through $k1$) to have dropped to 10 V when the capacity $C_{j+1}(t) \approx 0$ (similarly for other voltages). As may be seen, large voltage drops do yield marked reductions in range and endurance due to the increase in required current to maintain power. The voltage drops presented are somewhat larger than is usually seen during discharge. Although it would be desirable to explicitly validate the derived relations using reliable data, suitable material for comparison could not be located.

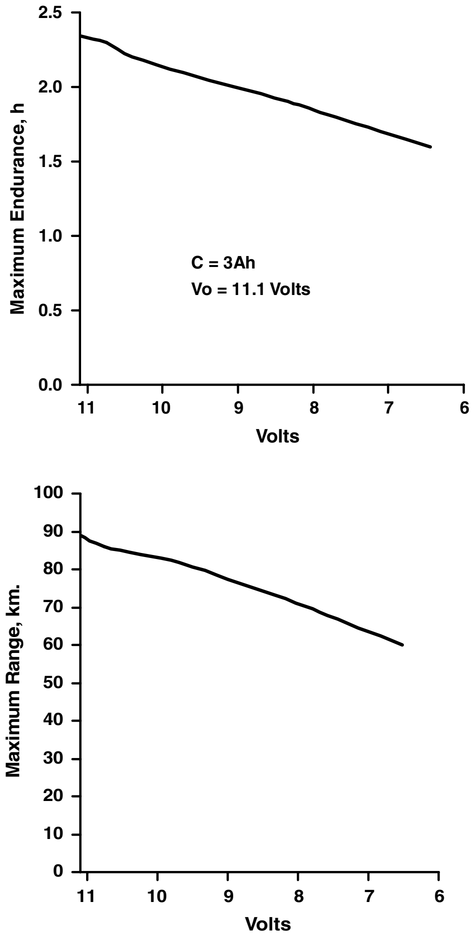

Fig. 4 Effect of battery discharge voltage on maximum range and endurance.

It should be mentioned that, for small UAVs, achieving initial altitude and flight velocity may require high current draw. As such, the initial capacity used for range and endurance estimates may be less than that for which the battery is rated. A recent study [10] has indicated that the effective capacity of lithium–ion–polymer batteries is approximately independent of their discharge rate. This is indicative of the behavior of an ideal battery. While this result is not to the author's knowledge substantiated/supported by additional studies, it may be readily implemented in the presented methodology by setting $n = 1$.

Conclusions

Relations are developed to estimate the range and endurance of a battery-powered aircraft. The effects of the battery discharge rate, as well as voltage drop, on effective capacity are examined. It is shown that the so-called Peukert effect can increase the range and endurance of a vehicle if the battery capacity is large with respect to the current required. Conversely, effective capacity is reduced if the current draw is close to the batteries' nominal capacity. For a constrained geometry and a fixed battery weight, as a fraction of the total aircraft weight, increasing battery capacity reduces performance due to greater required power and, consequently, current draw (which outweighs the capacity increase).

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